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INTERVAL EXTENSION STRUCTURE BASIC TYPES OF SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR LINEAR PARTIAL DIFFERENTIAL EQUATIONS OF THE FOURTH ORDER

Muminov B. B., Nazirov Sh. A.

Abstract. In this article, the interval expansion of the structure of solving basic types of boundary value problems for partial differential equations of the fourth order has been developed. Used Method of R-functions for constructed coordinate sequences. Constructing interval extensions of structural formulas, we consider problems (1) on the transverse bending of thin plates and 5 problems on a plate - rigidly clamped plate, loosely supported plate, elastically fixed plates, partially rigidly clamped and partially elastically fixed plates, plates, partially rigidly clamped and partially free. For the problem, the rigidly clamped plate Formula (7) is an interval structure for solving the boundary value problem (4). Here $L = \{ -\omega, -\psi, \omega, -\psi, -\omega, \psi, \omega, \psi \}$, $L_1 = \{ -\omega, D_1, -\varphi, \omega, D_1, -\varphi, -\omega, D_1, \varphi, \omega, D_1, \varphi \}$, $L_2 = \{ -\omega^2, -\Phi, \omega^2, -\Phi, -\omega^2, \Phi, \omega^2, \Phi \}$, $[-\Phi, \Phi]$ is an indefinite interval function. For the free-supported plate problem, a solution is obtained for the interval expansion of the structure in the form (15), (17), $[-(\Phi_1), (\Phi_1)]$, $[-(\Phi_2), (\Phi_2)]$ - indefinite interval function, D_2, T_2 - differential operators of the form (11) and (12). For the problem of elastically fixed plates, a solution was obtained in the interval expansion of the structure in the form (21) - (24), $[-(\Phi_1), (\Phi_1)]$, $[-(\Phi_2), (\Phi_2)]$ - indefinite interval functions, D_2, T_2, D_1 - differential operators of the form (11) and (12), (3). For the problem of partially rigidly clamped and partially elastically fixed plates, a solution was obtained in the interval expansion of the structure in the form of (28), (30), (32), $[-(\Phi_1), (\Phi_1)]$, $[-(\Phi_2), (\Phi_2)]$ - indefinite interval functions, D_2, T_2, D_1 - differential operators of the form (11) and (12), (6). For the plate problem, partially rigidly pinched and partially free, a solution is obtained in the interval extension of the structure (40), (41), (42), $[-(\Phi_1), (\Phi_1)]$, $[-(\Phi_2), (\Phi_2)]$ - indefinite interval functions, D_2, T_2, D_1, D_3 - differential operators of the form (11), (12), (6) and (38).

Keywords: structure solutions, interval arithmetic, boundary value problems, R-function method.

Introduction

R-function method is used particularly for solving boundary value problems of mathematical physics, expressed differential, integral and integro-differential equations, which describes the mathematical models of various physical, technical, mechanical, etc. Process [1]. R-functions method allows us to construct coordinate sequences satisfying the boundary conditions exactly, without any approximations. However, when solving systems of differential (integro-differential) equations of high order due to bad of a matrix (full matrix of large orders, compiled as a result of sampling in the spatial variables using the method of R-functions) lost accuracy of the approximate solutions. Furthermore, such a loss of accuracy arise in cases where the initial data of the problem are not accurate, approximate values of the integrals are computed and error methods for solving governing equations, etc. These disadvantages can be eliminated by using interval method [2-4]. Hence the need to develop an algorithm combining R-functions method and interval method for solving practical problems, which we call the interval-valued R-function [5]. Therefore, we construct interval extensions for structural formulas constructed for the main types of boundary value problems transverse bending of thin plates, which is described by the differential equation in partial derivatives of the fourth order.

II. Constructing interval extensions of structural formula

For simplicity, we consider the problem of transverse bending of thin plates [9].

$$D \Delta^2 u = q \quad (1)$$

where, $D = \frac{Eh^3}{12(1-\nu^2)}$ - flexural rigidity plate, h - thickness of the plate, E - young's modulus, ν - poisons' ratio, q - transverse load, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, u - deflection plate.

In problems of transverse bending of thin plates on the boundary Γ (or portions Γ_i) is defined by the two boundary conditions.

1. Rigidly clamped plate. On the plate contour is zero deflection and rotation angles:

$$u \Big|_{\Gamma} = 0, \quad \frac{\partial u}{\partial n} \Big|_{\Gamma} = 0 \quad (2)$$

Here, the normal n to Γ in this case can be used the following structural formula [1]:

$$u = \omega^2 \Phi \quad (3)$$

It is assumed that $|\nabla \omega| > 0$ on Γ . In practice there are options of rigid support with transverse displacements and rotations of the plate edge. For him, instead of the boundary conditions (2) are the conditions

$$u \Big|_{\Gamma} = \varphi_0, \quad \frac{\partial u}{\partial n} \Big|_{\Gamma} = \psi_0 \quad (4)$$

where, φ_0, ψ_0 - given functions on Γ . Denote the extension to Γ as $EC\varphi_0 = \varphi, EC\psi_0 = \psi$, (EC - operator bonding boundary values) will continue in Γ and also the structure of the solution of (1), (3) building type polynomial R-functions method has the form [1]:

$$u = \varphi + \omega\psi - \omega D_1\varphi + \omega^2\Phi \quad (5)$$

Where, Φ - undefined function, D_1 - differential operator with the following species,

$$D_1 u = \sum_{i=1}^n \frac{\partial \omega}{\partial x_i} \frac{\partial u}{\partial x_i}, \quad (6)$$

and, ω - equation of the boundary region, the following species

$$\omega = \begin{cases} > 0, & (x, y) \in \Omega \\ = 0, & (x, y) \in \Gamma, \\ < 0 & (x, y) \in \Gamma \cup \Omega \end{cases}$$

Applying interval arithmetic operations, construct interval extension solution structure (5):

$$\begin{aligned} [\underline{u}, \bar{u}] &= [\underline{\varphi}, \bar{\varphi}] + [\underline{\omega}, \bar{\omega}] [\underline{\psi}, \bar{\psi}] - [\underline{\omega}, \bar{\omega}] D_1 [\underline{\varphi}, \bar{\varphi}] + [\underline{\omega}, \bar{\omega}]^2 [\underline{\Phi}, \bar{\Phi}] = \\ &= [\underline{\varphi}, \bar{\varphi}] + [\min\{L\}, \max\{L\}] - [\min\{L_1\}, \max\{L_1\}] + [\min\{L_2\}, \max\{L_2\}] = \\ &= [\underline{\varphi} + \min\{L\} - \max\{L_1\} + \min\{L_2\}, \bar{\varphi} + \max\{L\} - \min\{L_1\} + \max\{L_2\}] \end{aligned}$$

and get

$$[\underline{u}, \bar{u}] = [\underline{\varphi} + \min\{L\} - \max\{L_1\} + \min\{L_2\}, \bar{\varphi} + \max\{L\} - \min\{L_1\} + \max\{L_2\}]. \quad (7)$$

Equation (7) is the interval structure of the solution of the problem (4).

Here

$$L = \{\underline{\omega}\psi, \bar{\omega}\psi, \underline{\omega}\bar{\psi}, \bar{\omega}\bar{\psi}\}, L_1 = \{\underline{\omega}D_1\varphi, \bar{\omega}D_1\varphi, \underline{\omega}D_1\bar{\varphi}, \bar{\omega}D_1\bar{\varphi}\}$$

$$L_2 = \{\underline{\omega}^2\Phi, \bar{\omega}^2\Phi, \underline{\omega}^2\bar{\Phi}, \bar{\omega}^2\bar{\Phi}\}, \quad [\underline{\Phi}, \bar{\Phi}] \text{ - undefined interval function.}$$

2. Simply supported plate. For her, the boundary conditions have the form

$$u|_{\Gamma} = 0, \quad \left(\frac{\partial^2 u}{\partial n^2} + v_0 \frac{\partial^2 u}{\partial \tau^2} \right) \Big|_{\Gamma} = 0 \quad (8)$$

where, n_0 and τ - direction of the normal and tangential to the Γ respectively. As shown in [1], this condition can also be written as

$$u|_{\Gamma} = 0, \quad \left(\frac{\partial^2 u}{\partial n^2} + \frac{v_0}{\rho} \frac{\partial u}{\partial \tau} \right) \Big|_{\Gamma} = 0 \quad (9)$$

where, τ - radius of curvature of Γ .

Structure of the solution of (1), (9) the construction method of the form R-polynomial function has the form [1]:

$$u = \omega\Phi_1 - \frac{\omega^2}{2} [D_2(\omega\Phi_1) + v_0 T_2(\omega\Phi_1) - \omega\Phi_2] \quad (10)$$

where, Φ_1, Φ_2 - undefined function and D_2, T_2 - differential operators, with the following form [1],

$$D_2 u = \left(\sum_{i=1}^n \frac{\partial \omega}{\partial x_i} \frac{\partial}{\partial x_i} \right)^2 u = \sum_{|a|=2} \frac{2}{a!} \prod_{i=1}^n \left(\frac{\partial \omega}{\partial x_i} \right)^{a_i} \frac{\partial^{|a|} u}{\partial x_1^{a_1} \dots \partial x_n^{a_n}} \quad (11)$$

$$T_2 u = \sum_{i=0}^2 (-1)^{2-i} C_m^i \frac{\partial^2 u}{\partial x_1^{m-i} \partial x_2^i} \left(\frac{\partial \omega}{\partial x_1} \right)^i \left(\frac{\partial \omega}{\partial x_2} \right)^{m-i} \quad (12)$$

Applying interval arithmetic operations, construct interval extension solution structure (9):

$$[\underline{u}, \bar{u}] = [\underline{\omega}, \bar{\omega}] [\underline{\Phi}_1, \bar{\Phi}_1] - \frac{[\underline{\omega}, \bar{\omega}]^2}{2} \times \\ \times [D_2([\underline{\omega}, \bar{\omega}][\underline{\Phi}_1, \bar{\Phi}_1]) + v_0 T_2([\underline{\omega}, \bar{\omega}][\underline{\Phi}_1, \bar{\Phi}_1]) - ([\underline{\omega}, \bar{\omega}][\underline{\Phi}_2, \bar{\Phi}_2])]. \quad (13)$$

taking into account that

$$[\underline{\omega}, \bar{\omega}] [\underline{\Phi}_1, \bar{\Phi}_1] = [\underline{X}_1, \bar{X}_1], \quad \text{where } \underline{X}_1 = \min\{X_1\}, \bar{X}_1 = \max\{X_1\}, \\ X_1 = \{\underline{\omega}\Phi_1, \underline{\omega}\bar{\Phi}_1, \bar{\omega}\Phi_1, \bar{\omega}\bar{\Phi}_1\},$$

$$\frac{[\underline{\omega}, \bar{\omega}]^2}{2} = [\underline{a}_1, \bar{a}_1], \quad \text{where } \underline{a}_1 = \min\{a_1\}, \bar{a}_1 = \max\{a_1\}, \\ a_1 = \left\{ \frac{1}{2} \underline{\omega}^2, \frac{1}{2} \bar{\omega}^2 \right\},$$

$$D_2([\underline{\omega}, \bar{\omega}][\underline{\Phi}_1, \bar{\Phi}_1]) = [\underline{\omega}, \bar{\omega}] D_2[\underline{\Phi}_1, \bar{\Phi}_1] + [\underline{\Phi}_1, \bar{\Phi}_1] D_2[\underline{\omega}, \bar{\omega}] = [\underline{X}_2, \bar{X}_2], \\ \text{where } [\underline{\omega}, \bar{\omega}] D_2[\underline{\Phi}_1, \bar{\Phi}_1] = [\underline{X}_5, \bar{X}_5], [\underline{\Phi}_1, \bar{\Phi}_1] D_2[\underline{\omega}, \bar{\omega}] = [\underline{X}_6, \bar{X}_6], \\ \underline{X}_5 = \min\{X_5\}, \bar{X}_5 = \max\{X_5\}, \\ X_5 = \{\underline{\omega}D_2\Phi_1, \underline{\omega}D_2\bar{\Phi}_1, \bar{\omega}D_2\Phi_1, \bar{\omega}D_2\bar{\Phi}_1\}, \\ \underline{X}_6 = \min\{X_6\}, \bar{X}_6 = \max\{X_6\},$$

$$X_6 = \{\underline{\Phi}_1 D_2 \underline{\omega}, \bar{\Phi}_1 D_2 \underline{\omega}, \underline{\Phi}_1 D_2 \bar{\omega}, \bar{\Phi}_1 D_2 \bar{\omega}\}, \\ [\underline{X}_2, \bar{X}_2] = [\underline{X}_5, \bar{X}_5] + [\underline{X}_6, \bar{X}_6] = [\underline{X}_5 + \underline{X}_6, \bar{X}_5 + \bar{X}_6], \\ v_0 T_2([\underline{\omega}, \bar{\omega}][\underline{\Phi}_1, \bar{\Phi}_1]) = v_0 [\underline{\omega}, \bar{\omega}] T_2[\underline{\Phi}_1, \bar{\Phi}_1] + v_0 [\underline{\Phi}_1, \bar{\Phi}_1] T_2[\underline{\omega}, \bar{\omega}] = [\underline{X}_3, \bar{X}_3], \\ \text{where } v_0 [\underline{\omega}, \bar{\omega}] T_2[\underline{\Phi}_1, \bar{\Phi}_1] = [\underline{X}_7, \bar{X}_7], v_0 [\underline{\Phi}_1, \bar{\Phi}_1] T_2[\underline{\omega}, \bar{\omega}] = [\underline{X}_8, \bar{X}_8], \\ \underline{X}_7 = \min\{X_7\}, \bar{X}_7 = \max\{X_7\},$$

$$X_7 = \{v_0 \underline{\omega} T_2 \Phi_1, v_0 \underline{\omega} T_2 \bar{\Phi}_1, v_0 \bar{\omega} T_2 \Phi_1, v_0 \bar{\omega} T_2 \bar{\Phi}_1\},$$

$$\underline{X}_6 = \min\{X_6\}, \quad \overline{X}_6 = \max\{X_6\},$$

$$X_6 =$$

$$\{v_0 \underline{\Phi}_1 D_2 \underline{\omega}, v_0 \overline{\Phi}_1 D_2 \underline{\omega}, v_0 \underline{\Phi}_1 D_2 \overline{\omega}, v_0 \overline{\Phi}_1 D_2 \overline{\omega}\},$$

$$[\underline{\omega}, \overline{\omega}] [\underline{\Phi}_2, \overline{\Phi}_2] = [\underline{X}_4, \overline{X}_4],$$

$$\text{where } \underline{X}_4 = \min\{X_4\}, \quad \overline{X}_4 = \max\{X_4\},$$

$$X_4 = \{\underline{\omega} \underline{\Phi}_2, \underline{\omega} \overline{\Phi}_2, \overline{\omega} \underline{\Phi}_2, \overline{\omega} \overline{\Phi}_2\},$$

and obtain

$$[\underline{u}, \overline{u}] = [\underline{X}_1, \overline{X}_1] - [\underline{a}_1, \overline{a}_1] \left[[\underline{X}_2, \overline{X}_2] + [\underline{X}_3, \overline{X}_3] - [\underline{X}_4, \overline{X}_4] \right]. \quad (14)$$

taking, into account that

$$[\underline{X}_2, \overline{X}_2] + [\underline{X}_3, \overline{X}_3] - [\underline{X}_4, \overline{X}_4] = [\underline{X}_9, \overline{X}_9],$$

where

$$[\underline{X}_9, \overline{X}_9] = [\underline{X}_2 + \underline{X}_3 - \overline{X}_4, \overline{X}_2 + \overline{X}_3 - \underline{X}_4],$$

$$[\underline{a}_1, \overline{a}_1] [\underline{X}_9, \overline{X}_9] = [\underline{X}_{10}, \overline{X}_{10}],$$

$$\text{where } \underline{X}_{10} = \min\{X_{10}\}, \quad \overline{X}_{10} = \max\{X_{10}\},$$

$$X_{10} = \{\underline{a}_1 \underline{X}_9, \underline{a}_1 \overline{X}_9, \overline{a}_1 \underline{X}_9, \overline{a}_1 \overline{X}_9\},$$

and obtain

$$[\underline{u}, \overline{u}] = [\underline{X}_1, \overline{X}_1] - [\underline{X}_{10}, \overline{X}_{10}] = [\underline{X}_1 - \overline{X}_{10}, \overline{X}_1 - \underline{X}_{10}] \quad (15)$$

or

$$[\underline{u}, \overline{u}] = [\underline{X}_1, \overline{X}_1] - [\underline{a}_1, \overline{a}_1] [\underline{X}_2, \overline{X}_2] - [\underline{a}_1, \overline{a}_1] [\underline{X}_3, \overline{X}_3] + [\underline{a}_1, \overline{a}_1] [\underline{X}_4, \overline{X}_4]. \quad (16)$$

taking, into account that

$$[\underline{a}_1, \overline{a}_1] [\underline{X}_2, \overline{X}_2] = [\underline{X}_{11}, \overline{X}_{11}],$$

$$\text{where } \underline{X}_{11} = \min\{X_{11}\}, \quad \overline{X}_{11} = \max\{X_{11}\},$$

$$X_{11} = \{\underline{a}_1 \underline{X}_2, \underline{a}_1 \overline{X}_2, \overline{a}_1 \underline{X}_2, \overline{a}_1 \overline{X}_2\},$$

$$[\underline{a}_1, \overline{a}_1] [\underline{X}_3, \overline{X}_3] = [\underline{X}_{12}, \overline{X}_{12}],$$

$$\text{where } \underline{X}_{12} = \min\{X_{12}\}, \quad \overline{X}_{12} = \max\{X_{12}\},$$

$$X_{12} = \{\underline{a}_1 \underline{X}_3, \underline{a}_1 \overline{X}_3, \overline{a}_1 \underline{X}_3, \overline{a}_1 \overline{X}_3\},$$

$$[\underline{a}_1, \overline{a}_1] [\underline{X}_4, \overline{X}_4] = [\underline{X}_{13}, \overline{X}_{13}],$$

$$\text{Where } \underline{X}_{13} = \min\{X_{13}\}, \quad \overline{X}_{13} = \max\{X_{13}\},$$

$$X_{13} = \{\underline{a}_1 \underline{X}_4, \underline{a}_1 \overline{X}_4, \overline{a}_1 \underline{X}_4, \overline{a}_1 \overline{X}_4\},$$

and obtain

$$[\underline{u}, \overline{u}] = [\underline{X}_1, \overline{X}_1] - [\underline{X}_{11}, \overline{X}_{11}] - [\underline{X}_{12}, \overline{X}_{12}] + [\underline{X}_{13}, \overline{X}_{13}] =$$

$$= [\underline{X}_1 - \overline{X}_{11} - \overline{X}_{12} + \underline{X}_{13}, \overline{X}_1 - \underline{X}_{11} - \underline{X}_{12} + \overline{X}_{13}] \quad (17)$$

In the interval extension structure (15), (17),

$$[\underline{\Phi}_1, \overline{\Phi}_1], [\underline{\Phi}_2, \overline{\Phi}_2] - \text{undefined gap function, } D_2, T_2$$

- differential operators of the form (11) and (12).

3. Elastically fixed plate. For them deflection function u satisfies the boundary conditions on Γ

$$u|_{\Gamma} = 0, \quad (18)$$

$$\left(\frac{\partial^2 u}{\partial n^2} + \frac{\partial^2 u}{\partial \tau^2} + \left(\frac{1 - \nu_0}{\rho_0} + k_0 \right) \frac{\partial u}{\partial n} \right) |_{\Gamma} = 0 \quad (19)$$

where ρ_0 - the radius of curvature of the boundary Γ , a k_0 - flexural rigidity of termination.

Structure of the solution of (1), (18) and (19) the construction method of the form R-polynomial function has the form [1]:

$$u = \omega \Phi_1 + \frac{1}{2} \omega^3 \Phi_2 - \frac{1}{2} \omega^2 [\Phi_1 (D_2 \omega + \nu_0 T_2 \omega + k) + 2D_1 \Phi_1] \quad (20)$$

where Φ_1, Φ_2 - undefined function and D_2, T_2, D_1 - differential operators of the form (11) and (12), (6).

Applying interval arithmetic operations, construct interval arithmetic structure of the solution (20):

$$[\underline{u}, \overline{u}] = [\underline{\omega}, \overline{\omega}] [\underline{\Phi}_1, \overline{\Phi}_1] + \frac{1}{2} [\underline{\omega}, \overline{\omega}]^3 [\underline{\Phi}_2, \overline{\Phi}_2] -$$

$$- \frac{1}{2} [\underline{\omega}, \overline{\omega}]^2 \left[[\underline{\Phi}_1, \overline{\Phi}_1] (D_2 [\underline{\omega}, \overline{\omega}] + \nu_0 T_2 [\underline{\omega}, \overline{\omega}] + k) + 2D_1 [\underline{\Phi}_1, \overline{\Phi}_1] \right] \quad (21)$$

considering that

$$[\underline{\omega}, \overline{\omega}] [\underline{\Phi}_1, \overline{\Phi}_1] = [\underline{X}_1, \overline{X}_1], \quad \frac{1}{2} [\underline{\omega}, \overline{\omega}]^2 = [\underline{a}_1, \overline{a}_1],$$

$$\frac{1}{2} [\underline{\omega}, \overline{\omega}]^3 [\underline{\Phi}_2, \overline{\Phi}_2] = [\underline{X}_{14}, \overline{X}_{14}], \quad \text{where}$$

$$\underline{X}_{14} = \min\{X_{14}\}, \quad \overline{X}_{14} = \max\{X_{14}\},$$

$$X_{14} = \left\{ \frac{1}{2} \underline{\omega}^3 \underline{\Phi}_2, \frac{1}{2} \underline{\omega}^3 \overline{\Phi}_2, \frac{1}{2} \overline{\omega}^3 \underline{\Phi}_2, \frac{1}{2} \overline{\omega}^3 \overline{\Phi}_2 \right\},$$

and obtain (20) as follows:

$$[\underline{u}, \overline{u}] = [\underline{X}_1, \overline{X}_1] + [\underline{X}_{14}, \overline{X}_{14}] - [\underline{a}_1, \overline{a}_1]$$

$$\left[[\underline{\Phi}_1, \overline{\Phi}_1] (D_2 [\underline{\omega}, \overline{\omega}] + \nu_0 T_2 \omega, \nu_0 T_2 \overline{\omega}] + k \right) + [2D_1 \underline{\Phi}_1, 2D_1 \overline{\Phi}_1] \right] \quad (22)$$

or

$$[\underline{u}, \overline{u}] = [\underline{X}_1, \overline{X}_1] + [\underline{X}_{14}, \overline{X}_{14}] - [\underline{a}_1, \overline{a}_1] \left[[\underline{\Phi}_1, \overline{\Phi}_1] (D_2 [\underline{\omega}, \overline{\omega}] + \nu_0 T_2 \omega, \nu_0 T_2 \overline{\omega}] + k \right) + [\underline{a}_1, \overline{a}_1] [2D_1 \underline{\Phi}_1, 2D_1 \overline{\Phi}_1] \right]$$

$$[\underline{u}, \overline{u}] = [\underline{X}_1, \overline{X}_1] + [\underline{X}_{14}, \overline{X}_{14}] - [\underline{a}_1, \overline{a}_1] \left[[\underline{\Phi}_1, \overline{\Phi}_1] (D_2 [\underline{\omega}, \overline{\omega}] + \nu_0 T_2 \omega, \nu_0 T_2 \overline{\omega}] + k \right) + [\underline{a}_1, \overline{a}_1] [2D_1 \underline{\Phi}_1, 2D_1 \overline{\Phi}_1] \right] \quad (23)$$

Taking into account that

$$[\underline{a}_1, \overline{a}_1] [\underline{\Phi}_1, \overline{\Phi}_1] = [\underline{W}_1, \overline{W}_1],$$

$$\text{where } \underline{W}_1 = \min\{W_1\}, \quad \overline{W}_1 = \max\{W_1\},$$

$$W_1 = \{\underline{a}_1 \underline{\Phi}_1, \underline{a}_1 \overline{\Phi}_1, \overline{a}_1 \underline{\Phi}_1, \overline{a}_1 \overline{\Phi}_1\},$$

$$[\underline{W}_1, \overline{W}_1] [D_2 \underline{\omega}, D_2 \overline{\omega}] = [\underline{W}_2, \overline{W}_2],$$

$$\text{where } \underline{W}_2 = \min\{W_2\}, \quad \overline{W}_2 = \max\{W_2\},$$

$$W_2 = \{\underline{W}_1 D_2 \underline{\omega}, \underline{W}_1 D_2 \overline{\omega}, \overline{W}_1 D_2 \underline{\omega}, \overline{W}_1 D_2 \overline{\omega}\},$$

$$\begin{aligned} \left[\underline{W}_1, \overline{W}_1 \right] \left[v_0 T_2 \underline{\omega}, v_0 T_2 \overline{\omega} \right] &= \left[\underline{W}_3, \overline{W}_3 \right], \text{ where} \\ \underline{W}_3 &= \min\{W_3\}, \quad \overline{W}_3 = \max\{W_3\}, \\ W_3 &= \left\{ W_1 v_0 T_2 \underline{\omega}, W_1 v_0 T_2 \overline{\omega}, W_1 v_0 T_2 \underline{\omega}, W_1 v_0 T_2 \overline{\omega} \right\}, \\ \left[\underline{W}_1, \overline{W}_1 \right] k &= \left[k \underline{W}_1, k \overline{W}_1 \right] = \left[\underline{W}_4, \overline{W}_4 \right], \left[a_1, \overline{a}_1 \right] \left[2D_1 \underline{\phi}_1, 2D_1 \overline{\phi}_1 \right] = \left[\underline{W}_5, \overline{W}_5 \right], \\ \text{where } \underline{W}_5 &= \min\{W_5\}, \quad \overline{W}_5 = \max\{W_5\}, \\ W_5 &= \left\{ a_1 2D_1 \underline{\phi}_1, a_1 2D_1 \overline{\phi}_1, \overline{a}_1 2D_1 \underline{\phi}_1, \overline{a}_1 2D_1 \overline{\phi}_1 \right\}, \end{aligned}$$

and we obtain (23) as follows:

$$\begin{aligned} \left[\underline{u}, \overline{u} \right] &= \left[\underline{X}_1, \overline{X}_1 \right] + \left[\underline{X}_{14}, \overline{X}_{14} \right] - \left[\underline{W}_2, \overline{W}_2 \right] + \left[\underline{W}_3, \overline{W}_3 \right] + \left[\underline{W}_4, \overline{W}_4 \right] + \left[\underline{W}_5, \overline{W}_5 \right] = \\ &= \left[\underline{X}_1, \overline{X}_1 \right] + \left[\underline{X}_{14}, \overline{X}_{14} \right] - \left[\underline{W}_2, \overline{W}_2 \right] - \left[\underline{W}_3, \overline{W}_3 \right] - \left[\underline{W}_4, \overline{W}_4 \right] - \left[\underline{W}_5, \overline{W}_5 \right] = \\ &= \left[\underline{X}_1 + \underline{X}_{14} - \overline{W}_2 - \overline{W}_3 - \overline{W}_4 - \overline{W}_5, \overline{X}_1 + \overline{X}_{14} - \underline{W}_2 - \underline{W}_3 - \underline{W}_4 - \underline{W}_5 \right] \quad (24) \end{aligned}$$

In the interval extension structure (21), (24), $\left[\underline{\phi}_1, \overline{\phi}_1 \right], \left[\underline{\phi}_2, \overline{\phi}_2 \right]$ - undefined interval function, D_2, T_2, D_1 - differential operators of the form (11) and (12) (3).

4. Partially clamped rigidly fixed and partially elastic plate. Let the contour Γ plate consists of two disjoint parts of Γ_1 and Γ_2 on the first of which is rigidly clamped plate, and the second - resiliently mounted. The boundary conditions in this case are

$$\begin{aligned} \underline{u} \Big|_{\Gamma_1} &= 0; \quad \frac{\partial \underline{u}}{\partial n} \Big|_{\Gamma_1} = 0; \\ \underline{u} \Big|_{\Gamma_2} &= 0; \quad \left(\frac{\partial^2 \underline{u}}{\partial n^2} + \frac{\partial^2 \underline{u}}{\partial \tau^2} + \left(\frac{1-v_0}{\rho_0} + k_0 \right) \frac{\partial \underline{u}}{\partial n} \right) \Big|_{\Gamma_2} = 0, \quad (26) \end{aligned}$$

where, v_0, ρ_0, k_0 and Poisson's ratio are, respectively, the radius of curvature r_2 and stiffness fixing plate. As before, we denote $EC\rho_0^{-1} = \rho^{-1}$, $ECk_0 = k$ (EC - extension operator boundary values).

Structure of the solution of (1), (24) and (25) the construction method of the form R-polynomial function has the form [1]:

$$\underline{u} = \frac{\omega_1^2}{\omega_1^2 + \omega_2^2} \left\{ \omega_2 \underline{\phi}_1 + \omega_2^3 \underline{\phi}_2 - \frac{\omega_2^2}{2} \left[\underline{\phi}_1 \left(D_2^{(2)} \omega_2 + v_0 T_2^{(2)} \omega_2 + k \right) + 2D_1^{(2)} \underline{\phi}_1 \right] \right\} \quad (27)$$

where, $\underline{\phi}_1, \underline{\phi}_2$ - undefined function and D_2, T_2, D_1 - differential operators of the form (11) and (12), (6).

Applying interval arithmetic operations, construct interval arithmetic structure of the solution (27):

$$\left[\underline{u}, \overline{u} \right] = \frac{\left[\underline{\omega}_1, \overline{\omega}_1 \right]^2}{\left(\left[\underline{\omega}_1, \overline{\omega}_1 \right]^2 + \left[\underline{\omega}_2, \overline{\omega}_2 \right]^2 \right)} \left\{ \left[\underline{\omega}_2, \overline{\omega}_2 \right] \left[\underline{\phi}_1, \overline{\phi}_1 \right] + \left[\underline{\omega}_2, \overline{\omega}_2 \right]^3 \left[\underline{\phi}_2, \overline{\phi}_2 \right] - \right.$$

$$\begin{aligned} &\left. \frac{\left[\underline{\omega}_2, \overline{\omega}_2 \right]^2}{2} \left[\left[\underline{\phi}_1, \overline{\phi}_1 \right] \left(D_2^{(2)} \left[\underline{\omega}_2, \overline{\omega}_2 \right] + v_0 T_2^{(2)} \left[\underline{\omega}_2, \overline{\omega}_2 \right] + k \right) + 2D_1^{(2)} \left[\underline{\phi}_1, \overline{\phi}_1 \right] \right] \right\} \quad (28) \end{aligned}$$

or

$$\begin{aligned} \left[\underline{u}, \overline{u} \right] &= \frac{\left[\underline{\omega}_1, \overline{\omega}_1 \right]^2}{\left(\left[\underline{\omega}_1, \overline{\omega}_1 \right]^2 + \left[\underline{\omega}_2, \overline{\omega}_2 \right]^2 \right)} \left\{ \left[\underline{\omega}_2, \overline{\omega}_2 \right] \left[\underline{\phi}_1, \overline{\phi}_1 \right] + \left[\underline{\omega}_2, \overline{\omega}_2 \right]^3 \left[\underline{\phi}_2, \overline{\phi}_2 \right] - \right. \\ &\left. \frac{\left[\underline{\omega}_2, \overline{\omega}_2 \right]^2}{2} \left[D_2^{(2)} \left[\underline{\omega}_2, \overline{\omega}_2 \right] \left[\underline{\phi}_1, \overline{\phi}_1 \right] + v_0 T_2^{(2)} \left[\underline{\omega}_2, \overline{\omega}_2 \right] \left[\underline{\phi}_1, \overline{\phi}_1 \right] + k \left[\underline{\phi}_1, \overline{\phi}_1 \right] + \right. \right. \\ &\left. \left. + 2D_1^{(2)} \left[\underline{\phi}_1, \overline{\phi}_1 \right] \right] \right\} \quad (29) \end{aligned}$$

taking into account that

$$\frac{\left[\underline{\omega}_1, \overline{\omega}_1 \right]^2}{\left(\left[\underline{\omega}_1, \overline{\omega}_1 \right]^2 + \left[\underline{\omega}_2, \overline{\omega}_2 \right]^2 \right)} = \left[\underline{B}_1, \overline{B}_1 \right],$$

where $\underline{B}_1 = \min\{B_1\}, \overline{B}_1 = \max\{B_1\}$,

$$\begin{aligned} (28) &\rightarrow \left\{ \omega_1^2 \frac{1}{(\omega_1^2 + \omega_2^2)}, \omega_1^2 \frac{1}{(\omega_1^2 + \omega_2^2)}, \omega_1^2 \frac{1}{(\omega_1^2 + \omega_2^2)}, \omega_1^2 \frac{1}{(\omega_1^2 + \omega_2^2)} \right\} \\ &\left[\underline{\omega}_2, \overline{\omega}_2 \right] \left[\underline{\phi}_1, \overline{\phi}_1 \right] = \left[\underline{B}_2, \overline{B}_2 \right], \end{aligned}$$

where

$$\underline{B}_2 = \min\{B_2\}, \overline{B}_2 = \max\{B_2\}, B_2 = \left\{ \omega_2 \underline{\phi}_1, \omega_2 \overline{\phi}_1, \overline{\omega}_2 \underline{\phi}_1, \overline{\omega}_2 \overline{\phi}_1 \right\},$$

$$\left[\underline{\omega}_2, \overline{\omega}_2 \right]^3 \left[\underline{\phi}_2, \overline{\phi}_2 \right] = \left[\underline{B}_3, \overline{B}_3 \right],$$

where

$$\underline{B}_3 = \min\{B_3\}, \overline{B}_3 = \max\{B_3\}, B_3 = \left\{ \omega_2^3 \underline{\phi}_2, \omega_2^3 \overline{\phi}_2, \overline{\omega}_2^3 \underline{\phi}_2, \overline{\omega}_2^3 \overline{\phi}_2 \right\},$$

$$\frac{\left[\underline{\omega}_2, \overline{\omega}_2 \right]^2}{2} = \frac{\left[\underline{\omega}_2^2, \overline{\omega}_2^2 \right]}{2} = \left[\frac{\omega_2^2}{2}, \frac{\overline{\omega}_2^2}{2} \right] = \left[\underline{b}, \overline{b} \right], \quad D_2^{(2)} \left[\underline{\omega}_2, \overline{\omega}_2 \right] \left[\underline{\phi}_1, \overline{\phi}_1 \right] = \left[\underline{B}_4, \overline{B}_4 \right],$$

where

$$\underline{B}_4 = \min\{B_4\}, \overline{B}_4 = \max\{B_4\}, B_4 = \left\{ D_2^{(2)} \omega_2 \underline{\phi}_1, D_2^{(2)} \omega_2 \overline{\phi}_1, D_2^{(2)} \overline{\omega}_2 \underline{\phi}_1, D_2^{(2)} \overline{\omega}_2 \overline{\phi}_1 \right\},$$

$$v_0 T_2^{(2)} [\underline{\omega}_2, \overline{\omega}_2] [\underline{\phi}_1, \overline{\phi}_1] = [\underline{B}_5, \overline{B}_5],$$

where

$$\underline{B}_5 = \min\{B_5\}, \overline{B}_5 = \max\{B_5\}, B_5 = \left\{ \underline{A}_1 B_6, \underline{A}_1 \overline{B}_6, \overline{A}_1 B_6, \overline{A}_1 \overline{B}_6 \right\},$$

$$\left\{ v_0 T_2^{(2)} \underline{\omega}_2 \underline{\phi}_1, v_0 T_2^{(2)} \underline{\omega}_2 \overline{\phi}_1, v_0 T_2^{(2)} \overline{\omega}_2 \underline{\phi}_1, v_0 T_2^{(2)} \overline{\omega}_2 \overline{\phi}_1 \right\} [\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_7, \overline{B}_7] = [\underline{C}_6, \overline{C}_6],$$

$$k [\underline{\phi}_1, \overline{\phi}_1] = [k \underline{\phi}_1, k \overline{\phi}_1] = [\underline{B}_6, \overline{B}_6],$$

$$2D_1^{(2)} [\underline{\phi}_1, \overline{\phi}_1] = [2D_1^{(2)} \underline{\phi}_1, 2D_1^{(2)} \overline{\phi}_1] = [\underline{B}_7, \overline{B}_7],$$

and obtain (28) as follows:

$$[\underline{u}, \overline{u}] = [\underline{B}_1, \overline{B}_1] \left\{ [\underline{B}_2, \overline{B}_2] + [\underline{B}_3, \overline{B}_3] - [\underline{b}, \overline{b}] \left[[\underline{B}_4, \overline{B}_4] + [\underline{B}_5, \overline{B}_5] + [\underline{B}_6, \overline{B}_6] + [\underline{B}_7, \overline{B}_7] \right] \right\} \quad (30)$$

or

$$[\underline{u}, \overline{u}] = [\underline{B}_1, \overline{B}_1] [\underline{B}_2, \overline{B}_2] + [\underline{B}_1, \overline{B}_1] [\underline{B}_3, \overline{B}_3] - [\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_4, \overline{B}_4] - [\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_5, \overline{B}_5] - [\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_6, \overline{B}_6] - [\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_7, \overline{B}_7] \quad (31)$$

taking into account that

$$[\underline{B}_1, \overline{B}_1] [\underline{B}_2, \overline{B}_2] = [\underline{C}_1, \overline{C}_1],$$

where

$$\underline{C}_1 = \min\{C_1\}, \overline{C}_1 = \max\{C_1\}, C_1 = \left\{ \underline{B}_1 B_2, \underline{B}_1 \overline{B}_2, \overline{B}_1 B_2, \overline{B}_1 \overline{B}_2 \right\},$$

$$[\underline{B}_1, \overline{B}_1] [\underline{B}_3, \overline{B}_3] = [\underline{C}_2, \overline{C}_2],$$

where

$$\underline{C}_2 = \min\{C_2\}, \overline{C}_2 = \max\{C_2\}, C_2 = \left\{ \underline{B}_1 B_3, \underline{B}_1 \overline{B}_3, \overline{B}_1 B_3, \overline{B}_1 \overline{B}_3 \right\},$$

$$[\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_4, \overline{B}_4] = [\underline{C}_3, \overline{C}_3],$$

where

$$\underline{C}_3 = \min\{C_3\}, \overline{C}_3 = \max\{C_3\}, C_3 = \left\{ \underline{A}_1 B_4, \underline{A}_1 \overline{B}_4, \overline{A}_1 B_4, \overline{A}_1 \overline{B}_4 \right\},$$

$$\underline{A}_1 = \min\{A_1\}, \overline{A}_1 = \max\{A_1\}, A_1 = \left\{ \underline{B}_1 \underline{b}, \underline{B}_1 \overline{b}, \overline{B}_1 \underline{b}, \overline{B}_1 \overline{b} \right\},$$

$$[\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_5, \overline{B}_5] = [\underline{C}_4, \overline{C}_4],$$

where

$$\underline{C}_4 = \min\{C_4\}, \overline{C}_4 = \max\{C_4\}, C_4 = \left\{ \underline{A}_1 B_5, \underline{A}_1 \overline{B}_5, \overline{A}_1 B_5, \overline{A}_1 \overline{B}_5 \right\},$$

$$[\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_6, \overline{B}_6] = [\underline{C}_5, \overline{C}_5],$$

where

$$\underline{C}_5 = \min\{C_5\}, \overline{C}_5 = \max\{C_5\}, C_5 = \left\{ \underline{A}_1 B_6, \underline{A}_1 \overline{B}_6, \overline{A}_1 B_6, \overline{A}_1 \overline{B}_6 \right\},$$

where

$$\underline{C}_6 = \min\{C_6\}, \overline{C}_6 = \max\{C_6\}, C_6 = \left\{ \underline{A}_1 B_7, \underline{A}_1 \overline{B}_7, \overline{A}_1 B_7, \overline{A}_1 \overline{B}_7 \right\},$$

and obtain (30) as follows:

$$[\underline{u}, \overline{u}] = [\underline{C}_1, \overline{C}_1] + [\underline{C}_2, \overline{C}_2] - [\underline{C}_3, \overline{C}_3] - [\underline{C}_4, \overline{C}_4] - [\underline{C}_5, \overline{C}_5] - [\underline{C}_6, \overline{C}_6] \quad (32)$$

or

$$[\underline{u}, \overline{u}] = [\underline{C}_1 + \underline{C}_2 - \underline{C}_3 - \underline{C}_4 - \underline{C}_5 - \underline{C}_6, \overline{C}_1 + \overline{C}_2 - \overline{C}_3 - \overline{C}_4 - \overline{C}_5 - \overline{C}_6] \quad (33)$$

In the interval extension structure (28), (30), (32),

$$[\underline{B}_1, \overline{B}_1] [\underline{B}_2, \overline{B}_2] - [\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_4, \overline{B}_4] - [\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_5, \overline{B}_5] - [\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_6, \overline{B}_6] - [\underline{B}_1, \overline{B}_1] [\underline{b}, \overline{b}] [\underline{B}_7, \overline{B}_7] - \text{undefined interval function,}$$

D_2, T_2, D_1 - differential operators of the form (11) and (12) (3).

5. Plate partially clamped rigidly and partly free.

Let Γ_1 - Γ section of the border, in which the plate is rigidly clamped and the rest of the boundary $\Gamma_2 = \Gamma \setminus \Gamma_1$ - free. The boundary conditions in this case are

$$u \Big|_{\Gamma_1} = 0; \quad \frac{\partial u}{\partial n} \Big|_{\Gamma_1} = 0;$$

$$\left(\frac{\partial^2 u}{\partial n^2} + v_0 \frac{\partial^2 u}{\partial \tau^2} \right) \Big|_{\Gamma_2} = 0; \quad (35)$$

$$\left(\frac{\partial^3 u}{\partial n^3} + (2 - v_0) \frac{\partial^3 u}{\partial n \partial \tau^2} \right) \Big|_{\Gamma_2} = 0. \quad (36)$$

Structure of the solution of (1), (34), (35) and (36) the construction method of the form R-polynomial function has the form [1]:

$$u = \frac{\omega_1^2}{\omega_1^2 + \omega_2^2} \left\{ \omega_2^4 \phi_2 + \phi_1 - \frac{\omega_2^3}{6} \left[D_3^{(2)} + (2 - v_0) D_2^{(2)} \right] \left\{ \phi_1 - [\omega_2^2 D_2^{(2)} \phi_1 + v_0 T_2^{(2)} \phi_1] \right\} - \frac{\omega_2^3}{2} \left[D_2^{(2)} \phi_1 - v_0 T_2^{(2)} \phi_1 \right] \right\} \quad (37)$$

where ϕ_1, ϕ_2 - undefined function and D_2, T_2, D_1 - differential operators of the form (11) and (12), (3),

D_3 - differential operator with the following form [1],

$$D_3 u = \left(\sum_{i=1}^n \frac{\partial \omega}{\partial x_i} \frac{\partial}{\partial x_i} \right)^3 u = \sum_{|a|=3} \frac{3!}{a!} \prod_{i=1}^n \left(\frac{\partial \omega}{\partial x_i} \right)^{a_i} \frac{\partial^n u}{\partial x_1^{a_1} \dots \partial x_n^{a_n}} \quad (38)$$

and applying the interval arithmetic operations, construct interval extension solution structure (37):

$$\begin{aligned} [\underline{u}, \bar{u}] &= \frac{[\underline{\omega_1}, \bar{\omega_1}]^2}{[\underline{\omega_1}, \bar{\omega_1}]^2 + [\underline{\omega_2}, \bar{\omega_2}]^4} \\ &\left\{ \left[\underline{\omega_2}, \bar{\omega_2} \right]^4 \left[\underline{\phi_2}, \bar{\phi_2} \right] + \left[\underline{\phi_1}, \bar{\phi_1} \right] - \frac{[\underline{\omega_2}, \bar{\omega_2}]^3}{6} \left[D_3^{(2)} + (2-v_0) \left(D_1^{(2)} T_2^{(2)} \right) \right] \right. \\ &\left. \left\{ \left[\underline{\phi_1}, \bar{\phi_1} \right] - \left[\underline{\omega_2}, \bar{\omega_2} \right]^2 D_2^{(2)} \left(\left[\underline{\phi_1}, \bar{\phi_1} \right] \right) + v_0 \left[\underline{\omega_2}, \bar{\omega_2} \right]^2 T_2^{(2)} \left(\left[\underline{\phi_1}, \bar{\phi_1} \right] \right) \right\} - \right. \\ &\left. - \frac{[\underline{\omega_2}, \bar{\omega_2}]^3}{2} \left[D_2^{(2)} \left(\left[\underline{\phi_1}, \bar{\phi_1} \right] \right) - v_0 T_2^{(2)} \left(\left[\underline{\phi_1}, \bar{\phi_1} \right] \right) \right] \right\}. \end{aligned} \quad (39)$$

taking into account that

$$\begin{aligned} \frac{[\underline{\omega_1}, \bar{\omega_1}]^2}{[\underline{\omega_1}, \bar{\omega_1}]^2 + [\underline{\omega_2}, \bar{\omega_2}]^4} &= [\underline{V_1}, \bar{V_1}], \text{ where } \underline{V_1} = \min\{V_1\}, \bar{V_1} = \max\{V_1\}, \\ V_1 &= \left\{ \omega_1^2 \frac{1}{(\omega_1^2 + \omega_2^2)}, \omega_1^2 \frac{1}{(\omega_1^2 + \omega_2^2)}, \bar{\omega}_1^2 \frac{1}{(\omega_1^2 + \omega_2^2)}, \bar{\omega}_1^2 \frac{1}{(\omega_1^2 + \omega_2^2)} \right\}, \\ [\underline{\omega_2}, \bar{\omega_2}]^4 [\underline{\phi_2}, \bar{\phi_2}] &= [\underline{V_2}, \bar{V_2}], \end{aligned}$$

where

$$\begin{aligned} \underline{V_2} &= \min\{V_2\}, \bar{V_2} = \max\{V_2\}, V_2 = \\ &\left\{ \omega_2^4 \phi_2, \omega_2^4 \bar{\phi}_2, \bar{\omega}_2^4 \phi_2, \bar{\omega}_2^4 \bar{\phi}_2 \right\}, \end{aligned}$$

$$\begin{aligned} D_3^{(2)} \frac{[\underline{\omega_2}, \bar{\omega_2}]^3}{6} &= \left[D_3^{(2)} \frac{\omega_2^3}{6}, D_3^{(2)} \frac{\bar{\omega}_2^3}{6} \right] = [\underline{V_3}, \bar{V_3}], \\ (2-v_0) \left(D_1^{(2)} T_2^{(2)} \right) \frac{[\underline{\omega_2}, \bar{\omega_2}]^3}{6} &= \\ = \left[\frac{(2-v_0) \left(D_1^{(2)} T_2^{(2)} \right) \omega_2^3}{6}, \frac{(2-v_0) \left(D_1^{(2)} T_2^{(2)} \right) \bar{\omega}_2^3}{6} \right] &= [\underline{V_4}, \bar{V_4}], \end{aligned}$$

$$[\underline{\omega_2}, \bar{\omega_2}]^2 D_2^{(2)} \left(\left[\underline{\phi_1}, \bar{\phi_1} \right] \right) = [\underline{V_5}, \bar{V_5}],$$

where

$$\begin{aligned} \underline{V_5} &= \min\{V_5\}, \bar{V_5} = \max\{V_5\}, V_5 = \\ &\left\{ \omega_2^2 D_2^{(2)} \phi_1, \omega_2^2 D_2^{(2)} \bar{\phi}_1, \bar{\omega}_2^2 D_2^{(2)} \phi_1, \bar{\omega}_2^2 D_2^{(2)} \bar{\phi}_1 \right\}, \end{aligned}$$

$$v_0 [\underline{\omega_2}, \bar{\omega_2}]^2 T_2^{(2)} \left(\left[\underline{\phi_1}, \bar{\phi_1} \right] \right) = [\underline{V_6}, \bar{V_6}],$$

where

$$\underline{V_6} = \min\{V_6\}, \bar{V_6} = \max\{V_6\}, V_6 = \left\{ v_0 \omega_2^2 T_2^{(2)} \phi_1, v_0 \omega_2^2 T_2^{(2)} \bar{\phi}_1, v_0 \bar{\omega}_2^2 T_2^{(2)} \phi_1, v_0 \bar{\omega}_2^2 T_2^{(2)} \bar{\phi}_1 \right\},$$

$$\frac{[\underline{\omega_2}, \bar{\omega_2}]^3}{2} D_2^{(2)} \left(\left[\underline{\phi_1}, \bar{\phi_1} \right] \right) = [\underline{V_7}, \bar{V_7}],$$

where

$$\begin{aligned} \underline{V_7} &= \min\{V_7\}, \bar{V_7} = \max\{V_7\}, V_7 = \\ &\left\{ \frac{\omega_2^3}{2} D_2^{(2)} \phi_1, \frac{\omega_2^3}{2} D_2^{(2)} \bar{\phi}_1, \frac{\bar{\omega}_2^3}{2} D_2^{(2)} \phi_1, \frac{\bar{\omega}_2^3}{2} D_2^{(2)} \bar{\phi}_1 \right\}, \end{aligned}$$

$$v_0 \frac{[\underline{\omega_2}, \bar{\omega_2}]^3}{2} T_2^{(2)} \left(\left[\underline{\phi_1}, \bar{\phi_1} \right] \right) = [\underline{V_8}, \bar{V_8}],$$

where

$$\begin{aligned} \underline{V_8} &= \min\{V_8\}, \bar{V_8} = \max\{V_8\}, V_8 = \\ &\left\{ v_0 \frac{\omega_2^3}{2} T_2^{(2)} \phi_1, v_0 \frac{\omega_2^3}{2} T_2^{(2)} \bar{\phi}_1, v_0 \frac{\bar{\omega}_2^3}{2} T_2^{(2)} \phi_1, v_0 \frac{\bar{\omega}_2^3}{2} T_2^{(2)} \bar{\phi}_1 \right\}, \end{aligned}$$

and obtain (39) as follows:

$$\begin{aligned} [\underline{u}, \bar{u}] &= [\underline{V_1}, \bar{V_1}] \left\{ \left[\underline{V_2}, \bar{V_2} \right] + \left[\underline{\phi_1}, \bar{\phi_1} \right] - \left[\underline{V_3}, \bar{V_3} \right] - \left[\underline{V_4}, \bar{V_4} \right] \left\{ \left[\underline{\phi_1}, \bar{\phi_1} \right] - \left[\underline{V_5}, \bar{V_5} \right] - \right. \right. \\ &\left. \left. - \left[\underline{V_6}, \bar{V_6} \right] \right\} - \left[\underline{V_7}, \bar{V_7} \right] + \left[\underline{V_8}, \bar{V_8} \right] \right\} \end{aligned} \quad (40)$$

or

$$\begin{aligned} [\underline{u}, \bar{u}] &= [\underline{V_1}, \bar{V_1}] \left[\underline{V_2}, \bar{V_2} \right] + [\underline{V_1}, \bar{V_1}] \left[\underline{\phi_1}, \bar{\phi_1} \right] - [\underline{V_1}, \bar{V_1}] \left[\underline{V_3}, \bar{V_3} \right] - [\underline{V_1}, \bar{V_1}] \left[\underline{V_4}, \bar{V_4} \right] \left[\underline{\phi_1}, \bar{\phi_1} \right] \\ &+ [\underline{V_1}, \bar{V_1}] \left[\underline{V_4}, \bar{V_4} \right] \left[\underline{V_5}, \bar{V_5} \right] + \\ &+ [\underline{V_1}, \bar{V_1}] \left[\underline{V_4}, \bar{V_4} \right] \left[\underline{V_6}, \bar{V_6} \right] - [\underline{V_1}, \bar{V_1}] \left[\underline{V_7}, \bar{V_7} \right] + [\underline{V_1}, \bar{V_1}] \left[\underline{V_8}, \bar{V_8} \right] \end{aligned} \quad (41)$$

taking into account that

$$[\underline{V_1}, \bar{V_1}] [\underline{V_2}, \bar{V_2}] = [\underline{G_1}, \bar{G_1}],$$

where

$$\begin{aligned} \underline{G_1} &= \min\{G_1\}, \bar{G_1} = \max\{G_1\}, G_1 = \\ &\left\{ V_1 V_2, V_1 \bar{V}_2, \bar{V}_1 V_2, \bar{V}_1 \bar{V}_2 \right\}, \end{aligned}$$

$$[\underline{V_1}, \bar{V_1}] [\underline{\phi_1}, \bar{\phi_1}] = [\underline{G_2}, \bar{G_2}],$$

where

$$\begin{aligned} \underline{G_2} &= \min\{G_2\}, \bar{G_2} = \max\{G_2\}, G_2 = \\ &\left\{ V_1 \phi_1, V_1 \bar{\phi}_1, \bar{V}_1 \phi_1, \bar{V}_1 \bar{\phi}_1 \right\}, \end{aligned}$$

$$[\underline{V_1}, \bar{V_1}] [\underline{V_3}, \bar{V_3}] = [\underline{G_3}, \bar{G_3}],$$

where

$$\begin{aligned} \underline{G_3} &= \min\{G_3\}, \bar{G_3} = \max\{G_3\}, G_3 = \\ &\left\{ V_1 V_3, V_1 \bar{V}_3, \bar{V}_1 V_3, \bar{V}_1 \bar{V}_3 \right\}, \end{aligned}$$

$$[\underline{V_1}, \bar{V_1}] [\underline{V_4}, \bar{V_4}] [\underline{\phi_1}, \bar{\phi_1}] = [\underline{G_4}, \bar{G_4}],$$

where

$$\underline{G}_4 = \min\{G_4\}, \overline{G}_4 = \max\{G_4\}, G_4 = \{\underline{V}_1 \underline{M}_1, \underline{V}_1 \overline{M}_1, \overline{V}_1 \underline{M}_1, \overline{V}_1 \overline{M}_1\},$$

$$\underline{M}_1 = \min\{M_1\}, \overline{M}_1 = \max\{M_1\}, M_1 = \{V_4 \Phi_1, V_4 \overline{\Phi}_1, \overline{V}_4 \Phi_1, \overline{V}_4 \overline{\Phi}_1\},$$

$$[\underline{V}_1, \overline{V}_1][\underline{V}_4, \overline{V}_4][\underline{V}_5, \overline{V}_5] = [\underline{G}_5, \overline{G}_5],$$

where

$$\underline{G}_5 = \min\{G_5\}, \overline{G}_5 = \max\{G_5\}, G_5 = \{\underline{V}_1 \underline{M}_2, \underline{V}_1 \overline{M}_2, \overline{V}_1 \underline{M}_2, \overline{V}_1 \overline{M}_2\},$$

$$\underline{M}_2 = \min\{M_2\}, \overline{M}_2 = \max\{M_2\}, M_2 = \{V_4 \underline{V}_5, V_4 \overline{V}_5, \overline{V}_4 \underline{V}_5, \overline{V}_4 \overline{V}_5\},$$

$$[\underline{V}_1, \overline{V}_1][\underline{V}_4, \overline{V}_4][\underline{V}_6, \overline{V}_6] = [\underline{G}_6, \overline{G}_6],$$

where

$$\underline{G}_6 = \min\{G_6\}, \overline{G}_6 = \max\{G_6\}, G_6 = \{\underline{V}_1 \underline{M}_3, \underline{V}_1 \overline{M}_3, \overline{V}_1 \underline{M}_3, \overline{V}_1 \overline{M}_3\},$$

$$\underline{M}_3 = \min\{M_3\}, \overline{M}_3 = \max\{M_3\}, M_3 = \{V_4 \underline{V}_6, V_4 \overline{V}_6, \overline{V}_4 \underline{V}_6, \overline{V}_4 \overline{V}_6\},$$

$$[\underline{V}_1, \overline{V}_1][\underline{V}_7, \overline{V}_7] = [\underline{G}_7, \overline{G}_7],$$

where

$$\underline{G}_7 = \min\{G_7\}, \overline{G}_7 = \max\{G_7\}, G_7 = \{\underline{V}_1 \underline{V}_7, \underline{V}_1 \overline{V}_7, \overline{V}_1 \underline{V}_7, \overline{V}_1 \overline{V}_7\},$$

$$[\underline{V}_1, \overline{V}_1][\underline{V}_8, \overline{V}_8] = [\underline{G}_8, \overline{G}_8],$$

where

$$\underline{G}_8 = \min\{G_8\}, \overline{G}_8 = \max\{G_8\}, G_8 = \{\underline{V}_1 \underline{V}_8, \underline{V}_1 \overline{V}_8, \overline{V}_1 \underline{V}_8, \overline{V}_1 \overline{V}_8\},$$

and obtain (41) as follows:

$$[\underline{u}, \overline{u}] = [\underline{G}_1, \overline{G}_1] + [\underline{G}_2, \overline{G}_2] - [\underline{G}_3, \overline{G}_3] - [\underline{G}_4, \overline{G}_4] + [\underline{G}_5, \overline{G}_5] + [\underline{G}_6, \overline{G}_6] - [\underline{G}_7, \overline{G}_7] + [\underline{G}_8, \overline{G}_8] \quad (42)$$

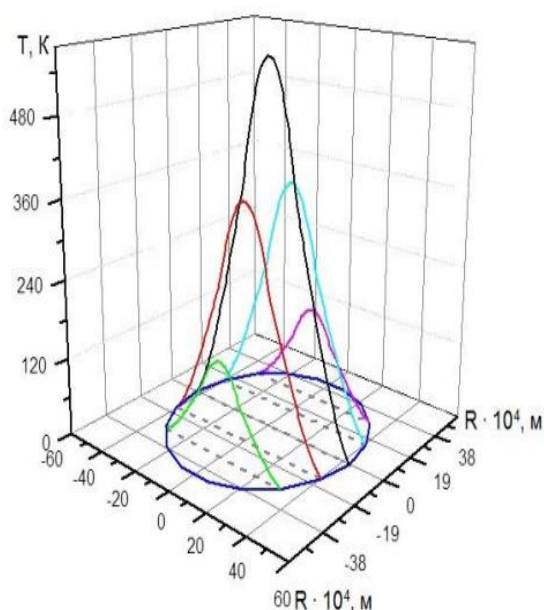
or

$$\underline{u} = \underline{G}_1 + \underline{G}_2 - \overline{G}_3 - \overline{G}_4 + \underline{G}_5 + \underline{G}_6 - \overline{G}_7 + \underline{G}_8$$

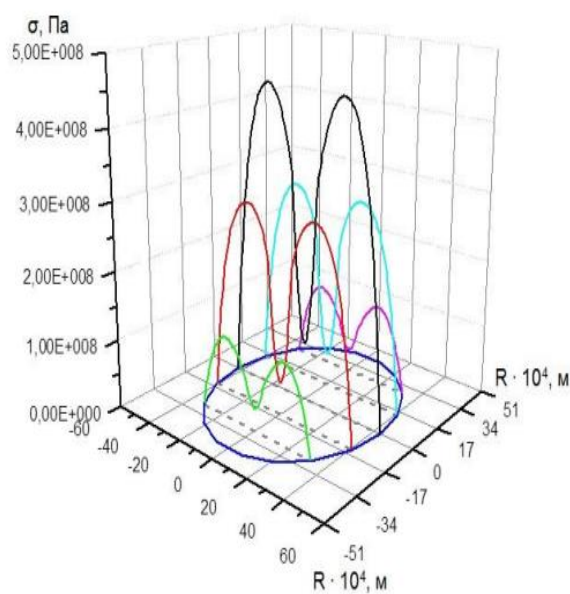
$$\overline{u} = \overline{G}_1 + \overline{G}_2 - \underline{G}_3 - \underline{G}_4 + \overline{G}_5 + \overline{G}_6 - \underline{G}_7 + \overline{G}_8 \quad (43)$$

In the interval extension structure (40), (41), (42), $[\underline{\Phi}_1, \overline{\Phi}_1], [\underline{\Phi}_2, \overline{\Phi}_2]$ - undefined interval function, D_2, T_2, D_1, D_3 - differential operators of the form (11), (12), (6) and (38).

When constructing interval extensions of structural formulas, we consider problems (1) on the transverse bending of thin plates and 5 problems on a plate — A rigidly clamped plate, A freely supported plate, Elastically fixed plates, Partially rigidly clamped and partially elastically fixed plates, Plates partially rigidly clamped and partially free and created software library "Sol4Interval" based on the object orientation of the C++ programming language. This library makes it possible to construct an interval 3D graph for interval expansion of structures (Fig-1).



a)



b)

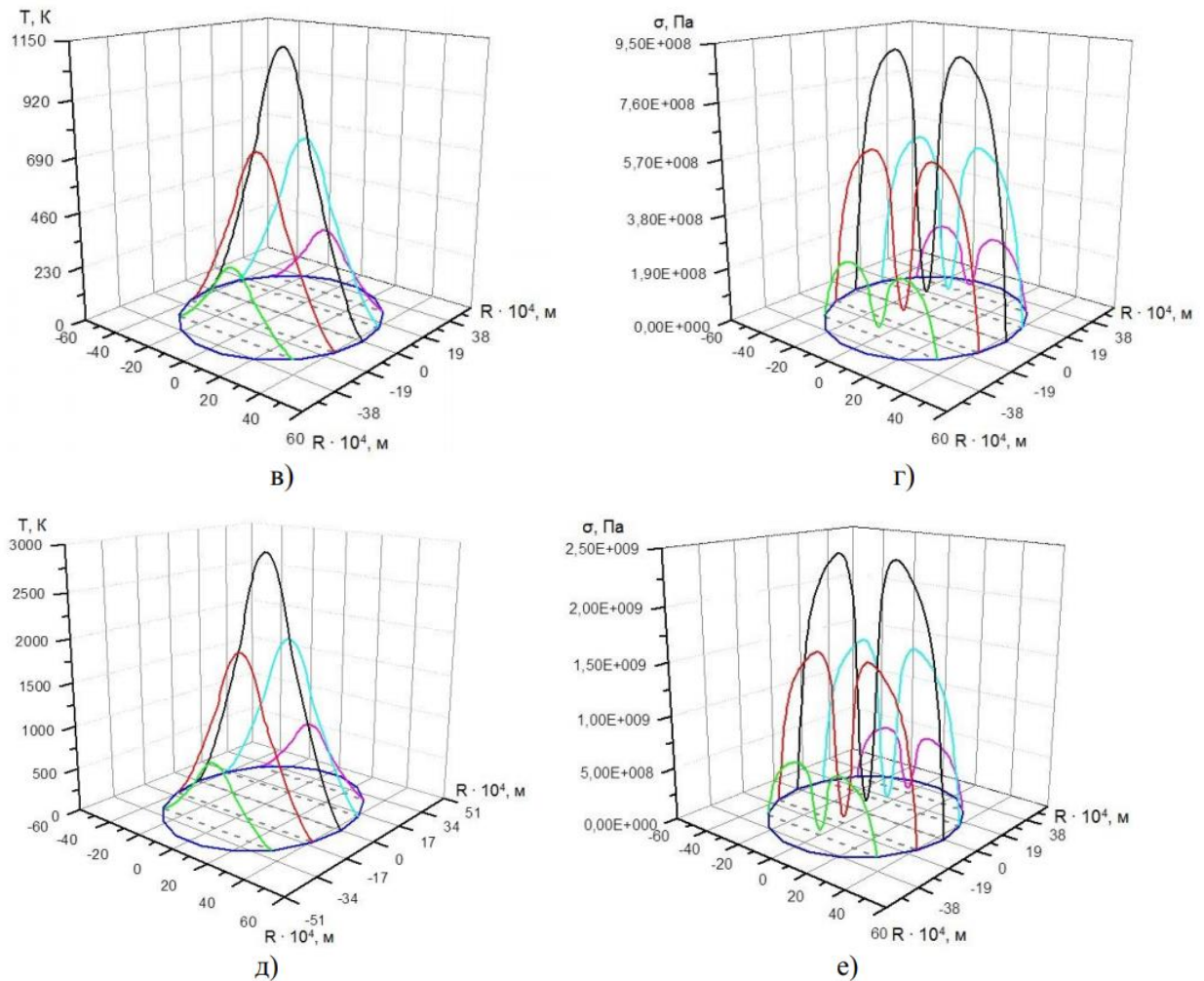


Figure 1. Distribution of temperatures (a, b, d) and temperature stresses (b, z, e) on surfaces $x = 4.54 \cdot 10^{-6}$ m (a, b), $1.44 \cdot 10^{-5}$ m (b, z) and $4.54 \cdot 10^{-5}$ m (d, e) under the action of ionizing radiation on a kiborite at a heat flux density of 1011 W/m^2 and its duration $t = 10^{-7}$ s (a, b), $t = 10^{-6}$ s (b, z) and $t = 10^{-5}$ s (d, e)

As a result of the calculations, the dependences (Fig. 1) of the maximum temperature in the zone of action of ionizing radiation on kiborite on the heat flux density at different depths in the body of the material and on the surface ($x = 0$) for different times of action were obtained.

There are three main areas for the successful use of interval analysis and interval methods on the basis of the Sol4Interval software library:

- Solving practical problems that have interval or, more generally, limited data uncertainty.
- Strict accounting of rounding errors in calculations with floating point numbers on digital computers.
- New approaches to solving traditional mathematical problems (such as, for example, the global optimization problem, global evidence-based solution of systems of nonlinear equations, etc.)

Conclusion

Interval extensions built structures making basic types of boundary value problems: rigidly clamped plate freely supported plate elastically supported plate partially clamped rigidly fixed and partially elastic

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plate partially clamped rigidly and partially free plate for differential equations of the fourth order. The results can be used to construct interval - value structure solutions for other boundary value problems of differential equations of different order. Resulting interval - values the solution structure is easily implemented in computers.

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